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Optimal Sanctions and Endogeneity of Differences in Detection Probabilities.

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Abstract

Offenders differ with respect to their detection probability in reality. Bebchuk and Kaplow (1993) conclude that sanctions should increase with the ability to avoid detection. We endogenize these differences in the ability to avoid detection and find that this can reverse the optimal sanction schedule. This paper thereby demonstrates how incentives for seemingly remote decisions can be manipulated through sanction structures.

Keywords: heterogeneous offenders, optimal law enforcement, education, detection probability

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1 Introduction

Heterogeneity of offenders is frequent in reality and can have drastic consequences for optimal enforcement policy. For example, Polinsky and Shavell (1991) consider varying wealth levels and find that the optimal fine equals total wealth for less well off and equals a defined level for offenders with higher wealth. Bebchuk and Kaplow (1993) introduce differences in detection probabilities. They consider different informational settings and find that if the enforcement authority: (i) can observe the type ex ante, it will tailor the sanction and the enforcement effort to each type, allowing the fine to be maximal; (ii) can observe the type ex post, it will be optimal to let fines increase in the difficulty of apprehension; and (iii) can determine the type neither ex ante nor ex post, the optimal fine may be below the maximal level and require an increase in enforcement effort.[1]

This paper scrutinizes the finding for the second informational context, which appeals because of the plausibility of the assumption concerning the information of the authority. Our analysis is triggered by the fact that policy makers may be ill-advised by the proposal of a sanction scheme that increases in the ability to avoid detection if determinants that are treated as exogenous in the derivation of the optimal policy are actually endogenous to the problem. That is why we endogenize the heterogeneity in detection probabilities by introducing an educational choice, in contrast to Bebchuk and Kaplow (1993) who take the differences in apprehension probabilities as exogenous. Differences in individuals' likelihood of avoiding detection can to a large extent be explained by education and learning (see e.g. Garoupa, forthcoming). We assume that education can be undertaken by every potential offender and causes detection probabilities to differ. However, society values education in general. There is a direct benefit to individuals in form of, for instance, higher self-valuation and job mobility. In addition, there are indirect benefits via synergies in the workplace, the stimulation of technological progress or increased political maturity, for instance.[2] Individuals' valuation of education may

[1]Different apprehension probabilities are taken up by Innes (2000) who finds that this realistic feature implies a further rationale for lower self-reporting sanctions. If the enforcement authority cannot observe the type at all, enforcement via unconditioned enforcement effort and sanctions is quite crude. In this context, self-reporting can prevent overdeterrence of easily apprehended individuals without affecting the deterrence of remaining potential offenders.

[2]See e.g. Moretti (2004) on externalities of education.

deviate from that of society due to both indirect benefits and the difference in apprehension probabilities, where the first is a social value not privately recognized, whereas the latter is only of private value. We find that sanction schemes may be employed to correct externalities in the education realm. In consequence, we find that allowing for education can reverse the result by Bebchuk and Kaplow (1993), i.e. that the optimal sanction scheme falls in the ability to avoid detection. The sanction scheme thus functions as a vehicle to internalize the positive externality of education. This results because the respective expected sanctions determine the partition of all individuals into offenders and non-offenders, and among offenders into those more difficult to apprehend because of the education obtained and those more easily apprehended. In this way, the policy maker's problem can to some extent be interpreted as one of marginal deterrence.[3] Marginal deterrence enters the picture in our model because undeterred offenders may offend being educated or uneducated. The policy maker therefore employs sanctions to provide incentives regarding (i) who should undertake the act at all and (ii) who should offend with education. As a consequence of our results, policy makers should be very wary when implementing sanction schemes that increase in the ability to avoid detection, for instance, by being more harsh when an act has been superiorly planned and conducted because of its potential effects on education.

In the literature concerning education and crime, Ehrlich (1975) was the first to empirically test relations of education and crime. More recently, Lochner (2004) finds empirical support for theoretically derived relationships with little bearing on our study, such as that older and more educated individuals commit fewer unskilled crimes. Lochner and Moretti (2004) empirically support the hypothesis that schooling reduces the probability of incarceration and arrest. In the sparse theoretical literature, a choice regarding education is considered in the search-theoretic framework of Huang, Laing and Wang (2004), for example. They allow for legitimate work and criminal activity and find - inter alia - that crime decreases the returns to human capital investments due to the possibility of theft. Usher (1997) assumes that education reduces the utility of any given

[3]Shavell (1992) considered the effect that sanctions and enforcement effort have on offenders if more than one offense is possible. In such a context, sanctions are of importance with respect to deterring individuals from committing any act but also with respect to which act undeterred offenders undertake.

offense in a general equilibrium model not at all concerned with optimal law enforcement. To our knowledge, our interest, that externalities in the education realm may be tackled with sanction schedules, has not been analyzed in the literature.

In section 2, we set out the model and, in our framework, reiterate the benchmark of Bebchuk and Kaplow (1993) without education, i.e. with exogenous differences in detection probabilities. We then introduce our generalization and highlight its effect on the optimal sanction scheme. Before we offer concluding remarks and end this study, results are illustrated in a numerical example. Proofs are relegated to the appendix.

2 The Model and Analysis

2.1 The Model

We extend the optimal-deterrence model extensively laid out in Garoupa (1997) or Polinsky and Shavell (2005). Risk-neutral individuals may commit an offense with social harm h. The benefit b of the act varies among individuals, with density function $f[b]$ and cumulative density function $F[b]$ on the support $[0, \infty]$.[4] We assume that the cumulative density functions used throughout are continuously differentiable. The policy maker does not observe b but knows the distribution. Social harm h is strictly less than infinity, so that some offenses are socially valuable. We allow for two different education levels θ, $\theta \in \{0, 1\}$. Benefit b is, for simplicity, independent from the attainment of education. We further assume that courts can observe the educational status θ of the offender by, for example, deducing the individual's craft from the conduct evident only after the act. Because of this ability ex post, courts can condition monetary sanctions according to the educational status, s^θ, $\theta = 0, 1$, whereas this does not apply to the enforcement effort x. Enforcement is thus general in Shavell's (1991) terminology because it cannot be tailored to acts of educated and uneducated individuals. This shrinks the set of different expected sanctions dependent on type θ from which the policy maker can choose. Sanctions have an upper boundary, the maximum sanction $\bar{s}$, which may be interpreted as the wealth of individuals. Enforcement effort x increases detection probability p at a

[4]Private equal social benefits of the act; see, e.g. Polinsky and Shavell (2005), footnote 8.

diminishing rate, $p_x > 0$, $p_{xx} < 0$. Besides, the detection probability p is a function of the type, i.e. educational status, of the offender, $p(x, \theta)$, $\theta = 0, 1$, with $p(x, 1) < p(x, 0)$ $\forall\, x$.[5] Increasing the detection probability uses resources x, whereas administrative costs of imposing the fine is a constant k, as in Polinsky and Shavell (1992). Education has social value δ. Potential offenders recognize only the share α of the social value, $\alpha\delta$, with $0 \leq \alpha \leq 1$. In addition, potential offenders value education because it reduces their detection probability. Furthermore, we assume that individuals bear a share β of the social costs k, $0 \leq \beta \leq 1$, because of psychic or opportunity costs of appearing before the enforcement authority, for instance. Consideration of imposition costs will allow us to contrast the effects of partial internalization of the value of education with that of imposition costs. The policy maker maximizes social welfare (to be more precisely defined below) with the knowledge of individuals' internalization shares α, β. For this purpose, the policy maker will set at least one of the two sanctions equal to the boundary $\bar{s}$. This can be expected because the exchange of sanction magnitude for detection probability magnitude for a given deterrence saves not only enforcement but also imposition costs and none of the arguments usually brought forth against maximal sanctions is present in our model.[6]

2.2 Exogenous Differences in Detection Probabilities

Assume an exogenous share q $(1-q)$ of potential offenders of type $\theta = 0$ (1). Individuals of type θ offend if the gain from doing so is greater than or equal to the total expected individual costs, $b \geq p(x, \theta)(s^\theta + \beta k)$. The total expected individual costs consist of the expected sanction and, if $\beta > 0$, of the expected individual costs due to the imposition costs of the sanction. The policy maker maximizes social welfare, which equals the benefits individuals derive from their acts and society derives from education less the harm done, administrative costs and enforcement effort. The Lagrangean with μ_θ as

[5] Lochner and Moretti (2004) resort to different data to deal with the effect of differences in apprehension probabilities due to varying educational achievements.

[6] This expectation is confirmed by our results spelled out in the appendix. None of the arguments against maximal sanctions are present in this framework except, of course, our focus, heterogeneous detection probabilities. Yet, this extension only reasons that some sanctions may be lower than the level possible. The other arguments against maximal sanctions are surveyed by Garoupa (1997), for instance.

Kuhn-Tucker multiplier reads

$$\begin{aligned}\max_{s^0,s^1,x} W =& q\int_{p(x,0)(s^0+\beta k)}^{\infty}(b-h-p(x,0)k)f[b]db \\ &+(1-q)\int_{p(x,1)(s^1+\beta k)}^{\infty}(b-h-p(x,1)k+\delta)f[b]db \\ &+(1-q)F[p(x,1)(s^1+\beta k)]\delta - x - \mu_0(s^0-\bar{s}) - \mu_1(s^1-\bar{s}) \end{aligned} \tag{1}$$

subject to $\mu_\theta \geq 0$ and $s^\theta \leq \bar{s}$, $\theta = 0, 1$.

Individuals of type $\theta = 0$ affect social welfare only if they commit a criminal act. In that case, benefit b is created, harm h suffered, and expected imposition costs $p(x,0)k$ arise. Individuals of type $\theta = 1$ affect social welfare in a similar way to that described for type $\theta = 0$ but in addition create value by their educational status. That is why individuals of type $\theta = 1$ whose benefit level does not make the commission of the act worthwhile, $b < p(x,1)(s^1 + \beta k)$, still affect social welfare by δ, the social value of education.

Taking the shares of educated and uneducated offenders as exogenously given, we find that the optimal sanction scheme is either flat or increasing in the ability to avoid detection, i.e. either $s^0 = s^1 = \bar{s}$ or $s^0 < s^1 = \bar{s}$ holds.[7] Thus, we obtain the result by Bebchuk and Kaplow (1993) in our broader framework as well. This result is quite plausible as the policy maker sets sanctions to deter crime efficiently without concern for other decisions by the individual. The policy maker may thus let sanctions increase in the ability to avoid detection to level expected sanctions of both types.

2.3 Endogenous Differences in Detection Probabilities

The difference in apprehension probability is now no longer exogenously determined but is due to education. All individuals are initially of type $\theta = 0$. Education costs τ vary among individuals according to density function $g[\tau]$ and cumulative density function $G[\tau]$ on the support $[0, \infty]$. Costs τ are for simplicity independent from benefit b.

Allowing for all possible combinations of respective sanctions could provoke the difference in total expected individual costs $\{s^0 p(x,0) - s^1 p(x,1) + \beta k[p(x,0) - p(x,1)]\}$ to

[7] Proof available upon request.

be (i) greater than or equal to zero and (ii) negative. In the following, we can constrain our argumentation to the first case. We do not discuss the case in which the difference turns negative because such a scheme would have no positive effect concerning deterrence but would discourage education. If the difference in total expected individual costs turns out to be zero in the optimum, we have a sanction scheme that increases in the ability to avoid detection, $s^0 < s^1$.[8] In effect, all individuals, uneducated and educated, would in that case consider the same borderline benefit as necessary for the commission of the act to be advantageous. Moreover, all individuals would choose education by comparing education costs with the private value $\alpha\delta$. A further change in the sanctions so that the difference becomes negative would, from a welfare perspective, negatively affect the offense and the education decision.

We now turn to the partition of individuals according to their benefit and education cost level. Among potential offenders, there are individuals whose benefit from the act is large enough to make it individually advantageous irrespective of education.[9] A subset of these individuals obtains education. Cut-off level $\eta = s^0p(x,0) - s^1p(x,1) + \beta k[p(x,0) - p(x,1)] + \alpha\delta$ details all of the potential advantages of education for individuals who offend irrespective of education, that is, (i) the difference in expected sanctions, (ii) a decrease in the probability of bearing personal administrative expenses, and (iii) the individually perceived value of education. Another set of individuals might offend only given the opportunity of education. The benefit of the act itself would not suffice in relation to the total expected costs of the act if undertaken uneducated. Finally, individuals for which $\tau \leq \alpha\delta$ and $b < \max\{(s^0 + \beta k)p(x,0); (s^1 + \beta k)p(x,1)\}$ holds, split into offenders and non-offenders, whereas all obtain education because the private value $\alpha\delta$ suffices in relation to their education costs τ.

From a policy maker's point of view, individuals that offend should choose education if $\tau \leq k[p(x,0) - p(x,1)] + \delta$, whereas individuals that do not offend should choose education as long as $\tau \leq \delta$. With respect to the benefit of the act, optimality is achieved if only offenses are undertaken for which $b \geq h + p(x,\theta)k$ holds. The policy maker takes

[8]This is true because $\{s^0p(x,0) + \beta kp(x,0) = s^1p(x,1) + \beta kp(x,1)\}$ requires $s^0 < s^1$ because of our assumption $p(x,0) > p(x,1)\ \forall\ x$.

[9]For these individuals, it holds that $b \geq \max\{(s^0 + \beta k)p(x,0), (s^1 + \beta k)p(x,1)\}$.

these goals into account. First, some offenders should rather offend with education, and, second, the decrease in detection probability due to education should not cheapen the offense too much because this would turn too many low-benefit individuals into offenders who would not offend without the opportunity given by education to lower detection probabilities.

The policy maker who takes all of these considerations into account solves the constrained problem

$$\begin{aligned}\max_{s^0,s^1,x} W =& F[(\beta k + s^1)p(x,1)] \int_0^{\alpha\delta} (\delta - \tau) g[\tau] d\tau \\ &+ [1 - G[\eta]] \int_{p(x,0)(s^0+\beta k)}^{\infty} (b - h - kp(x,0)) f[b] db \\ &+ \int_{\alpha\delta}^{\eta} \int_{\tau + p(x,1)(s^1+\beta k) - \alpha\delta}^{\infty} (b - h - \tau - kp(x,1) + \delta) f[b] db g[\tau] d\tau \\ &+ \int_0^{\alpha\delta} \int_{p(x,1)(s^1+\beta k)}^{\infty} (b - h - \tau - kp(x,1) + \delta) f[b] db g[\tau] d\tau \\ &- x - \mu_0(s^0 - \bar{s}) - \mu_1(s^1 - \bar{s}) \end{aligned} \tag{2}$$

subject to $\mu_\theta \geq 0$ and $s^\theta \leq \bar{s}$, $\theta = 0, 1$, and $\eta = s^0 p(x,0) - s^1 p(x,1) + \beta k[p(x,0) - p(x,1)] + \alpha\delta$.

In this social welfare function, we consider the benefit and harm from acts, the social value of education less the education costs, and the enforcement as well as expected imposition costs. Individuals may obtain education for the private benefit of education, $\alpha\delta$, alone or combine it with the transgression.

The Kuhn-Tucker conditions except $\mu_\theta \geq 0$ and $s^\theta \leq \bar{s}$, $\theta = 0, 1$, are:

(i) with respect to x

$$\begin{aligned}&\left\{\beta k[p_x(x,0) - p_x(x,1)] + s^0 p_x(x,0) - s^1 p_x(x,1)\right\} \left(1 - F[(\beta k + s^0)p(x,0)]\right) g[\eta]\psi \\ &+ (1 - G[\eta]) p_x(x,0) \left\{f[(\beta k + s^0)p(x,0)]\chi(\beta k + s^0) - k\left(1 - F[(\beta k + s^0)p(x,0)]\right)\right\} \\ &+ G[\alpha\delta] p_x(x,1) \left\{f[(\beta k + s^1)p(x,1)]\epsilon(\beta k + s^1) - k\left(1 - F[(\beta k + s^1)p(x,1)]\right)\right\} \\ &+ \int_{\alpha\delta}^{\eta} g[\tau] p_x(x,1) \left\{f[\gamma]\epsilon(\beta + s^1) - k(1 - F[\gamma])\right\} d\tau = 1 \end{aligned} \tag{3}$$

(ii) with respect to s^0

$$\begin{aligned} &p(x,0)\chi[1-G[\eta]]f[p(x,0)(s^0+\beta k)] \\ &+p(x,0)g[\eta]\psi\left(1-F[p(x,0)(s^0+\beta k)]\right)=\mu_0 \end{aligned} \tag{4}$$

(iii) with respect to s^1

$$\begin{aligned} &p(x,1)\epsilon[\int_{\alpha\delta}^{\eta} f[\gamma]g(\tau)d\tau+G[\alpha\delta]f[(\beta k+s^0)p(x,0)]] \\ &-p(x,1)g[\eta]\psi\left(1-F[p(x,0)(s^0+\beta k)]\right)=\mu_1 \end{aligned} \tag{5}$$

and finally (iv)

$$\mu_0(s^0-\bar{s})=0 \tag{6}$$

$$\mu_1(s^1-\bar{s})=0 \tag{7}$$

with $\psi=(1-\alpha)\delta+(1-\beta)k[p(x,0)-p(x,1)]-[s^0p(x,0)-s^1p(x,1)]$, which captures the difference between social and private benefits of education. If $\psi>0$ in the optimum, not all social benefits from education are reflected in the individual optimization, which provokes socially too little education. Education in excess of the social optimum occurs if $\psi<0$ holds. As $\chi=h+(1-\beta)kp(x,0)-s^0p(x,0)$ captures effects of the act not internalized by uneducated offenders, $\chi>0$ implies underdeterrence of uneducated offenders. The same holds for educated offenders and term $\epsilon=h-(1-\alpha)\delta+(1-\beta)kp(x,1)-s^1p(x,1)$. Finally, $\gamma=\tau+\beta kp(x,1)-\alpha\delta+s^1p(x,1)$ is the requirement with respect to the benefit level b for individuals that offend only given the opportunity of education. See figure 1 for a sketch of the partition of individuals.

In the figure, it is also possible to enumerate the effects of changing total expected costs from being equal to being different, i.e. the effects of moving line A to the left of line B. The respective areas stand for: (i) more crime without affecting the level of education, (ii) education is obtained only to offend, and (iii) more education results without changing crime.

The individual fully internalizes the social harm due to her offense and education decision if $s^0=h/p(x,0)+(1-\beta)k$ and $s^1=\frac{h-(1-\alpha)\delta}{p(x,1)}+(1-\beta)k$, because then the individual problem reflects the social problem, given some x.

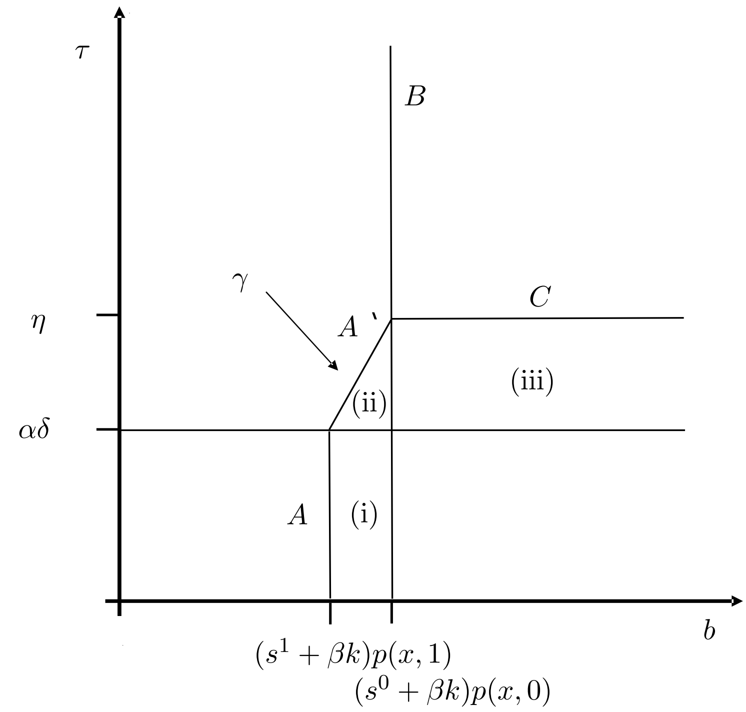

Figure 1: Respective combinations of education costs and benefit levels

Lemma 1 *In the optimum of the model of law enforcement with educational choice, neither expected sanction is equal to or greater than the harm due to the act, i.e. optimal enforcement entails some underdeterrence for acts of educated as well as uneducated offenders.*

What emerges in this enforcement model holds in the standard framework as well, namely that the endogeneity of the detection probability advocates expected sanctions less than the harm caused (see e.g. Polinsky and Shavell, 2005, 15-16).[10]

Now, we come to the core of our analysis. We analyze effects of endogenous education on the optimal sanction schedule in a model in which an externality may arise and thereby inquire whether the result of Bebchuk and Kaplow (1993) is robust to endogenizing differences in individuals' ability to avoid detection.

Proposition 1 *If differences in apprehension probabilities are the result of socially desirable education and*

[10]Shavell (1992) also obtains this result for his two-act model with general enforcement.

(i) individuals fully internalize the value of education and imposition costs ($\alpha = \beta = 1$), *then the optimal sanction scheme is either flat or increasing in the ability to avoid detection,*
(iia) individuals fully internalize the value of education but not imposition costs ($\alpha = 1 > \beta$), *then the optimal sanction scheme is either flat or increasing in the ability to avoid detection,*
(iib) individuals do not fully internalize the value of education but imposition costs ($\alpha < 1 = \beta$), *then the optimal sanction scheme can be decreasing, flat, or increasing in the ability to avoid detection,*
(iii) individuals neither internalize the value of education nor imposition costs fully ($\alpha, \beta < 1$), *then the optimal sanction scheme can be decreasing, flat, or increasing in the ability to avoid detection.*

A sanction scheme that decreases in the ability to avoid detection, $s^0 > s^1$, is a stimulus to further education. This stimulus is necessary if benefits internalized by the individual fall short of social benefits, i.e. $\psi > 0$ as stated above. In concurrence with this reasoning, we find that the policy maker encourages education by setting $s^0 > s^1$ only if $\psi > 0$ holds. Encouraging education through the sanction schedule is costly because it turns more individuals into offenders.

Interestingly, partial internalization of administrative costs k alone is not sufficient to make a decrease in the sanction scheme optimal, while partial internalization of δ is. This discrepancy is not clear at first sight, as $(1-\beta)k[p(x,0)-p(x,1)]$ is a positive share of the benefits of education that the individual does not internalize. In consequence, it could be expected that this just as well provides a reason to incentivize further education. We explain this feature by relating to the ideal sanctions that would result with exogenous enforcement effort and without an upper bound on sanctions already referred to above. If $\chi = h + (1-\beta)kp(x,0) - s^0 p(x,0) = 0$, uneducated offenders internalize all the harm they cause. From this follows $s^0 = h/p(x,0) + (1-\beta)k$. If $\epsilon = h - (1-\alpha)\delta + (1-\beta)kp(x,1) - s^1 p(x,1) = 0$, educated offenders internalize all the harm they cause and $s^1 = \frac{h-(1-\alpha)\delta}{p(x,1)} + (1-\beta)k$ can in that case be deduced. In that circumstance, the derivatives with respect to s^0 and s^1 are equal to zero because social benefits of education equal private benefits, i.e. $\psi = 0$. We can observe that whereas the partial internalization of the imposition costs is reflected symmetrically, the recognition of only a share α of the education impacts solely on the sanction of educated offenders. The sanctions that solve the more complex problem with maximal sanctions and endogenous enforcement are implicitly defined by the Kuhn-Tucker conditions stated above, yet the main principle given with reference to the ideal sanctions carries forward to that case and explains the divergent effects of the two different partial internalizations.

2.4 Numerical Example

For illustration, consider a simple example that confirms all of our findings of proposition 1. We take benefit b and education costs τ on the support $[0, 1]$ and assume for simplicity that both are uniformly distributed. Setting $h = .5$ leaves some offenses as indeed socially desirable. Moreover, as a benchmark we set $\delta = \alpha = \beta = .5$, $k = .125$, and for the maximum sanction $\bar{s} = 1$. With respect to the detection probability, we use the specification $p(x, \theta) = x^{.5}(\theta + 1)^{-1}$ that satisfies our assumptions $p_x > 0$, $p_{xx} < 0$, and $p(x, 0) > p(x, 1) \; \forall \; x$. With these inputs, the policy maker maximizes social welfare by the choice of $(s^0, s^1, x) = (1, .72, .023)$, i.e. if the sanction for educated offenders is less than that for uneducated offenders, $s^0 > s^1$. The optimum tends towards a flat sanction structure if δ (α) is decreased (increased). Starting from the benchmark, a flat scheme results if δ falls below .5 (α increases beyond .725). Furthermore, we find that with $\alpha = .5 < 1 = \beta$, that is with a partial internalization of educational value δ and a complete internalization of the imposition costs k, the optimal policy vector changes to $(s^0, s^1, x) = (1, .79, .024)$, i.e. a sanction scheme that decreases in the ability to avoid detection, $s^0 > s^1$, remains optimal. If we instead set $\beta = .5 < 1 = \alpha$, a decrease in the sanction schedule is no longer optimal. In that case, the optimal policy variables equal $(s^0, s^1, x) = (1, 1, .012)$.

3 Conclusion

Offenders differ in many respects. This variety also implies differences in individual apprehension probabilities. Bebchuk and Kaplow (1993) consider this fact and find that the optimal sanction scheme ought to increase with the individual's ability to avoid detection. We endogenize these differences in detection probabilities and find that this may reverse the optimal sanction scheme. Hence, allowing for the realistic endogeneity of differences in apprehension probabilities can reverse the policy proposal believed to be optimal hitherto. The rationale is that the sanction scheme can be used to incentivize individuals to obtain an education, which would otherwise not be obtained. The problem of the policy maker parallels one of marginal deterrence, where it also matters which act undeterred offenders undertake. The sanction scheme is consequently employed to correct externalities in the educational choice. This study thereby exemplifies that policy makers affect seemingly remote decisions by designing a sanction scheme. If potential offenders internalize the full social value of education, there is no need to employ the sanction scheme as incentive mechanism to further education. That is why, in that case, sanctions decreasing in the ability to avoid detection are no longer optimal. It is, however, very questionable to assume that individuals internalize the full value of education in reality. For instance, Usher (1997) enumerates as possible civic, fiscal, technical and altruistic externalities of education and Moretti (2004) establishes external effects empirically. Consequently, the case in which our framework yields the possibility of optimal sanctions decreasing in the ability to avoid detection is presumably most characteristic of reality.

Appendix

In the following, we search for optima that are consistent with the Kuhn-Tucker conditions of the policy maker's optimization problem, equations (3), (4), (5), (6), (7), and $\mu_\theta \geq 0$ as well as $s^\theta \leq \bar{s}$, $\theta = 0, 1$. In addition, our assumptions, in particular that concerning the respective detection probabilities, need to hold. As in Clements (2003), for instance, at this general level more than one scheme fulfills all necessary conditions for welfare maximization.

With respect to lemma 1: Proof. Let us define $A = (1-\alpha)\delta - (1-\beta)kp(x,1) + s^1 p(x,1)$ and $B = s^0 p(x,0) - (1-\beta)kp(x,0)$ and note that $\psi = A - B$, $\chi = h - B$ and $\epsilon = h - A$. First, we turn to the possibility that either sanction reflects expected harm:

(i) Suppose $s^0 = h/p(x,0) + (1-\beta)k$, that is $h = B$. This turns χ equal to zero. A can be (a) greater than, (b) equal to, or (c) less than h. (a) If $A > h = B$, it holds that $\psi > 0$ and $\epsilon < 0$. The derivative with respect to s^1, (5), yields a negative μ_1 in this case. (b) If $A = h = B$, the derivatives with respect to s^0 and s^1 are equal to zero. However, the condition with respect to x, equation (3), cannot be fulfilled. (c) If $A < h = B$, $\psi < 0$ together with $\chi = 0$ imply a negative value for μ_0 in the derivative with respect to s^0, (4). (ii) Suppose $s^1 = [h - (1-\alpha)\delta]/p(x,1) + (1-\beta)k$, i.e. $A = h$, which turns ϵ equal to zero. B can be (a) greater than, (b) equal to, or (c) less than h. (a) If $B > h = A$, it holds that $\psi < 0$ and $\chi < 0$. In this case, the derivative with respect to s^0, equation (4), yields a negative μ_0. (b) This constellation has been dealt with above. (c) If $B < h = A$, $\psi > 0$ together with $\epsilon = 0$ imply a negative value for μ_1 in the derivative with respect to s^1, (5). Thus, none of the sanctions equals harm to society not internalized by the offender.

We now turn to overdeterrence. (i) Suppose $s^0 > h/p(x,0) + (1-\beta)k$, that is $h < B$ which implies $\chi < 0$. A can be (a) greater than B, (b) equal to B, (c) less than B but greater than h, or (d) less than h (A is not equal to h was shown above). (a) If $A > B > h$, it holds that $\psi > 0$ and $\epsilon < 0$. Then, the term on the left-hand side of equation (5) is negative. (b) If $A = B > h$, we get $\psi = 0$ in combination with $\chi < 0$, which turns μ_0 negative in (4). (c) If $B > A > h$, $\mu_0 < 0$ results as $\psi < 0$ and $\chi < 0$. (d) If $B > h > A$, we again get $\psi < 0$ and $\chi < 0$, turning μ_0 in (4) negative. (ii) Suppose $s^1 > [h - (1-\alpha)\delta]/p(x,1) + (1-\beta)k$, that is $h < A$ which implies $\epsilon < 0$. B can be (a) greater than A, (b) equal to A, (c) less than A but greater than h, or (d) less than h (B is not equal to h was shown above). The only constellation we have not yet covered is (d). If $A > h > B$, it holds that $\psi > 0$ and $\epsilon < 0$. In this case, the derivative with respect to s^1 gives a negative μ_1. Hence, none of the sanctions is greater than the harm to society not accounted for in the individual optimization. ■

With respect to proposition 1: Proof. Below, we reason using the conditions with respect to s^0 and s^1. However, as the cases discussed below always include $h > A$ and $h > B$, implying $\chi > 0$ and $\epsilon > 0$, these cases thus can satisfy the first-order condition with regard to x. Equation (3) contains several terms of which, in the cases discussed below, at least three are strictly positive.

(i) We first consider the extreme case that potential offenders internalize costs k and benefit δ, i.e. $\alpha = \beta = 1$. Our terms defined above then reduce to $A = s^1 p(x,1)$ and $B = s^0 p(x,0)$. The optimum of the planner's problem is not characterized by either overdeterrence or expected sanction equal to harm, as was shown above. Then, we consider the residual cases of (a) $h > A > B$, (b) $h > A = B$, and (c) $h > B > A$. (a) This configuration turns $\psi > 0$, $\epsilon > 0$ and $\chi > 0$ and yields a positive μ_0 in (4) and μ_1 smaller than, equal to, or greater than zero in (5). Thus, this case looks at first sight as if it allows a decreasing sanction scheme. However, $A > B$ together with our assumption concerning apprehension probabilities, $p(x,0) > p(x,1)\ \forall x$, implies $s^0 < s^1$, which is inconsistent with $\mu_0 > 0$, and we therefore exclude this case. (b) $h > A = B$ together with $p(x,1)/p(x,0) < 1$ implies $s^0 < s^1$, which is inconsistent with the sign of the derivative with respect to s^0, which points at $s^0 = \bar{s}$ because of $\psi = 0$ and $\chi > 0$. Thus, this constellation is excluded. (c) $h > B > A$ implies $s^0/s^1 > p(x,1)/p(x,0)$ and thus would allow for a decreasing sanction scheme. However, μ_0 smaller than, equal to, or greater than zero and a positive μ_1 results for this case. Thus, possible outcomes are $s^0 \leq s^1 = \bar{s}$.

(ii) We now examine the cases in which either (a) $\alpha < \beta = 1$ or (b) $\beta < \alpha = 1$ holds, i.e. cases in which the potential offender either fully internalizes administrative costs or the educational value δ. (a) Terms A, B reduce to $A = (1-\alpha)\delta + s^1 p(x,1)$ and $B = s^0 p(x,0)$. We consider cases (a1) $h > A > B$, (a2) $h > A = B$, and (a3) $h > B > A$. (a1) This configuration yields a positive μ_0 in (4) and μ_1 smaller than, equal to, or greater than zero in (5). $A > B$ implies $p(x,1)/p(x,0) > s^0/s^1 - (1-\alpha)\delta/[s^1 p(x,0)]$. Thus, a decreasing sanction scheme, $s^0 = \bar{s} > s^1$, is consistent with this configuration and our assumption concerning the detection probability of the different types. A decreasing sanction scheme is more likely if the share of educational value internalized by the potential offender is low, which appeals intuitively. The sanction scheme $s^0 = s^1 = \bar{s}$ is another possibility. (a2) $h > A = B$ yields positive μ_0 and μ_1 in (4) and (5), respectively, and $A = B$ implies $p(x,1)/p(x,0) = s^0/s^1 - (1-\alpha)\delta/[s^1 p(x,0)]$, so that $s^0 = s^1 = \bar{s}$ is the only feasible solution. (a3) For the case that $h > B > A$,

$s^0 < s^1 = \bar{s}$ or $s^0 = s^1 = \bar{s}$ can result in the optimum. (b) After having examined the case that the value of education δ is only partly internalized whereas all administrative costs are taken into account by the individual, we now turn to the opposite case. A simplifies to $A = s^1 p(x,1) - (1-\beta)kp(x,1)$ and B to $B = s^0 p(x,0) - (1-\beta)kp(x,0)$. We again consider cases (b1) $h > A > B$, (b2) $h > A = B$, and (b3) $h > B > A$. (b1) $h > A > B$ implies $p(x,1)/p(x,0) > [s^0 - (1-\beta)k]/[s^1 - (1-\beta)k]$. As $p(x,1)/p(x,0) < 1$ by our assumptions, this constellation together with this assumption require $s^0 < s^1$. This, however, is inconsistent with the respective signs of the derivatives with respect to s^0 and s^1 as $\mu_0 > 0$ and μ_1 smaller than, equal to, or greater than zero imply that $s^0 = \bar{s} \geq s^1$. That is why we exclude this case. (b2) $A = B$ translates into $p(x,1)/p(x,0) = [s^0 - (1-\beta)k]/[s^1 - (1-\beta)k]$ and thereby requires $s^0 < s^1$ because of the assumption $p(x,1)/p(x,0) < 1$. However, assuming the specified ranking yields $\mu_0 > 0$ and $\mu_1 > 0$ which translates into $s^0 = s^1 = \bar{s}$. Thus, we exclude this case also. (b3) Reversing the ranking of A and B implies the condition $p(x,1)/p(x,0) < [s^0 - (1-\beta)k]/[s^1 - (1-\beta)k]$ which can principally be obtained with s^0 less than, equal to, or greater than s^1. However, taken together with the sign of the derivatives, the only feasible optimum entails $s^0 \leq s^1 = \bar{s}$.

(iii) Finally, we look at $\alpha, \beta < 1$. In this scenario, $A = (1-\alpha)\delta - (1-\beta)kp(x,1) + s^1 p(x,1)$ and $B = s^0 p(x,0) - (1-\beta)kp(x,0)$. Again, different possible constellations are (a) $h > A > B$, (b) $h > A = B$, and (c) $h > B > A$. (a) The condition resulting from $A > B$ together with the assumption regarding detection probabilities can hold for s^0 less, equal to, or, within limits, greater than s^1. However, the derivatives with respect to s^0 and s^1 narrow it down to two possible outcomes, $s^0 = \bar{s} \geq s^1$. Hence, this case can produce a decreasing sanction scheme. (b) The requirement that follows from $A = B$ together with the assumption regarding detection probabilities does not narrow possible optima. The derivatives, however, only allow for $s^0 = s^1 = \bar{s}$. (c) $h > B > A$: The derivatives indicate the possibility of an increasing or flat sanction scheme, i.e. $s^0 \leq s^1 = \bar{s}$ with no objection due to condition $B > A$ or our assumptions.

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References

Bebchuk, L.A. and L. Kaplow, 1993. Optimal Sanctions and Differences in Individuals' Likelihood of Avoiding Detection. *International Review of Law and Economics 13*, 217-224.

Clements, M., 2003. Precautionary Incentives for Privately Informed Victims. *International Review of Law and Economics 23*, 237-251.

Ehrlich, I., 1975. On the Relation between Education and Crime, in: Juster, F.T., *Education, Income, and Human Behavior*, McGraw-Hill.

Garoupa, N., forthcoming. An Economic Analysis of Criminal Law, in Hatzis, A. (ed.): *Economic Analysis of Law: A European Perspective*, Edward Elgar.

Garoupa, N., 1997. The Theory of Optimal Enforcement of Law. *Journal of Economics Surveys 11 (3)*, 267-295.

Huang, C., Laing, D. and P. Wang, 2004. Crime and Poverty: A Search-Theoretic Approach. *International Economic Review 45 (3)*, 909-39.

Innes, R., 2000. Self-Reporting in Optimal Law Enforcement When Violators Have Heterogeneous Probabilities of Apprehension. *Journal of Legal Studies 29*, 287-300.

Lochner, L., 2004. Education, Work, and Crime: A Human Capital Approach. *International Economic Review 45 (3)*, 811-43.

Lochner, L. and E. Moretti, 2004. The Effect of Education on Crime: Evidence from Prison Inmates, Arrests, and Self-Reports. *American Economic Review 94 (1)*, 155-89.

Moretti, E., 2004. Estimating the Social Return to Higher Education: Evidence from

Longitudinal and Repeated Cross-Sectional Data. *Journal of Econometrics 121*, 175-212.

Polinsky, A.M. and S. Shavell, 2005. The Theory of Public Enforcement of Law. *NBER Working Paper*, 11780.

Polinsky, A.M. and S. Shavell, 1992. Enforcement Costs and The Optimal Magnitude and Probability of Fines. *Journal of Law & Economics 35 (1)*, 133-148.

Polinsky, A.M. and S. Shavell, 1991. A Note on Optimal Fines When Wealth Varies Among Individuals. *American Economic Review 81 (3)*, 618-621.

Shavell, S., 1992. A Note on Marginal Deterrence. *International Review of Law and Economics 12*, 345-355.

Shavell, S., 1991. Specific versus General Enforcement of Law. *Journal of Political Economy 99 (5)*, 1088-1108.

Usher, D., 1997. Education as a Deterrent to Crime. *Canadian Journal of Economics 30 (2)*, 367-384.